

# Mark Scheme (Results)

Summer 2022

Pearson Edexcel International Advanced Level In Pure Mathematics P2 (WMA12) Paper 01

# **Edexcel and BTEC Qualifications**

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at <u>www.edexcel.com</u> or <u>www.btec.co.uk</u>. Alternatively, you can get in touch with us using the details on our contact us page at <u>www.edexcel.com/contactus</u>.

#### Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2022 Question Paper Log Number P71377A Publications Code WMA12\_01\_2206\_MS All the material in this publication is copyright © Pearson Education Ltd 2022

## **General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL IAL MATHEMATICS

## **General Instructions for Marking**

- 1. The total number of marks for the paper is 75.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:

#### <u>'M' marks</u>

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct

e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned. e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

#### <u>'A' marks</u>

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

#### <u>'B' marks</u>

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

# 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol  $\sqrt{}$  will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- **\*** The answer is printed on the paper
- The second mark is dependent on gaining the first mark
  - 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  - 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  - 6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
  - 7. Ignore wrong working or incorrect statements following a correct answer.

#### **General Principles for Pure Mathematics Marking**

(But note that specific mark schemes may sometimes override these general principles)

#### Method mark for solving 3 term quadratic:

1. Factorisation

 $(x^{2} + bx + c) = (x + p)(x + q)$ , where |pq| = |c|, leading to x = ... $(ax^{2} + bx + c) = (mx + p)(nx + q)$ , where |pq| = |c| and |mn| = |a|, leading to x = ...

#### 2. <u>Formula</u>

Attempt to use <u>correct</u> formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving  $x^2 + bx + c = 0$ :  $(x \pm \frac{b}{2})^2 \pm q \pm c$ ,  $q \neq 0$ , leading to x = ...

# Method marks for differentiation and integration:

#### 1. Differentiation

Power of at least one term decreased by 1. ( $x^n \rightarrow x^{n-1}$ )

2. Integration

Power of at least one term increased by 1. ( $x^n \rightarrow x^{n+1}$ )

#### Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

**Method mark** for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

#### **Exact answers**

Examiners' reports have emphasised that where, for example, an <u>exact</u> answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

#### Answers without working

The rubric says that these <u>may</u> not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does <u>not</u> cover this, please contact your team leader for advice.

Question	Scheme	Marks
1	1024 +	B1
	$\dots + {}^{10}C_1 \times \dots \times + {}^{10}C_2 \times \dots \times {}^2 + {}^{10}C_3 \times \dots \times {}^3 + \dots$	M1
	+10×2 <sup>9</sup> × $\frac{3}{8}x$ +45×2 <sup>8</sup> × $\left(\frac{3}{8}x\right)^2$ +120×2 <sup>7</sup> × $\left(\frac{3}{8}x\right)^3$ Or two of +1920x+1620x <sup>2</sup> +810x <sup>3</sup>	A1
	$\ldots + 1920x + 1620x^2 + 810x^3$	A1
		(4)
		(4 marks)

#### Notes:

B1: Correct constant term of 1024 as an integer.

M1: Correct binomial coefficient multiplied by the correct powers of x for at least 2 terms in x,  $x^2$  or  $x^3$ .

Allow e.g.  ${}^{10}C_1$ ,  ${}^{10}C_2$ ,  ${}^{10}C_3$  or  $\begin{pmatrix}10\\1\end{pmatrix}$ ,  $\begin{pmatrix}10\\2\end{pmatrix}$ ,  $\begin{pmatrix}10\\3\end{pmatrix}$  or evaluated coefficients and condone missing brackets e.g.  $\frac{3}{8}x^2$  for  $\left(\frac{3}{8}x\right)^2$  or  $\frac{3}{8}x^3$  for  $\left(\frac{3}{8}x\right)^3$ 

May take out a common factor of  $2^{10}$  first, but again look for correct binomial coefficients multiplied by the correct powers of x for **at least 2 terms** in x,  $x^2$  or  $x^3$ .

E.g. 
$$\left(2+\frac{3}{8}x\right)^{10} = 2^{10}\left(1+\frac{3}{16}x\right)^{10} = 2^{10}\left(1+{}^{10}C_1\dots x+{}^{10}C_2\dots x^2+{}^{10}C_3\dots x^3+\dots\right)$$

A1: Allow for a fully correct unsimplified expression (ignoring the constant term) with evaluated binomial coefficients, must be expanded if a common factor of  $2^{10}$  is taken out first **OR** two of the three terms in *x*,  $x^2$  and  $x^3$  correct and simplified.

E.g. ... + 10×2<sup>9</sup>×
$$\frac{3}{8}x$$
 + 45×2<sup>8</sup>× $\left(\frac{3}{8}x\right)^2$  + 120×2<sup>7</sup>× $\left(\frac{3}{8}x\right)^3$   
Or ... + 10×2<sup>10</sup>× $\frac{3}{16}x$  + 45×2<sup>10</sup>× $\left(\frac{3}{16}x\right)^2$  + 120×2<sup>10</sup>× $\left(\frac{3}{16}x\right)^3$ 

The brackets must be present unless they are implied by subsequent work. OR two of  $\dots + 1920x + 1620x^2 + 810x^3$ . Allow terms to be "listed".

# A1: Final three terms fully correct and simplified. Allow terms to be "listed".Once a correct expansion (or list of terms) is seen then isw. E.g. some candidates think they have to list the coefficients separately but apply isw.Ignore any extra terms if found.

For reference incorrect bracketing:  
...+
$$45 \times 2^8 \times \frac{3}{8}x^2 + 120 \times 2^7 \times \frac{3}{8}x^3$$
 gives ...+ $4320x^2 + 5760x^3$ 

And usually scores B1M1A0A0 if the 1024 is correct

<u>Special case:</u> Some candidates are just finding the coefficients 1024 + 1920 + 1620 + 810 and this scores B1 only if the *x*'s never make an appearance.

Question				Scheme				Marks
2(a)	x	0.5	1	1.5	2	2.5	3	B1
	У	1.319	1.075	1.001	1.041	1.223	1.850	B1
								(2)
(b)			h = 0	.5 seen or 11	nplied			BI
	Area	$\approx \left\{\frac{0.5}{2}\right\} \times \left(\frac{1}{2}\right)$	1.319+2("1	.075"+1.00	01+"1.041"	+1.223)+1	.850)	M1
			:	= awrt 2.90	5			A1
								(3)
(c)		3+	$\log_{10}(\sin(x))$	(1) = 4 - (1 - 1)	$-\log_{10}(\sin(x))$	x)))		B1
		$\int_{0.5}^{3} 3 + \log_{10}$	$(\sin(x)) dx$	$= [4x]_{0.5}^3 - '$	'2.96"=12	-2-"2.96		M1
		=	7.04 follow	w through 10	) – their 2.9	06		A1ft
								(3)
Alt (c)		$\int_{0.5}^{3} \log_{10}($	$(\sin(x)) dx =$	$=\int_{0.5}^{3}1\mathrm{d}x-\mathrm{d}x$	'2.96" = 2.5	5-"2.96"		B1
	$\int_{0.5}^{3} 3+$	$\log_{10}(\sin(x))$	)) dx = [3x]	$^{3}_{0.5}$ + (2.5 – "	2.96") = 9-	-1.5+2.5-	-"2.96"	M1
		=	7.04 follow	w through 10	) – their 2.9	96		A1ft
						(3)		
								(8 marks)
Notes:								
<ul> <li>(a)</li> <li>B1: Either of</li> <li>B1: Both of a</li> <li>Remember to</li> <li>Note that the</li> <li>(Which gives</li> </ul>	awrt 1.075 c awrt 1.075 an to check the l e incorrect va s 4.51 in (b))	or awrt.1.04 d awrt.1.041 body of the alues of 2.75	l correct correct script for tl 58 and 2.45	ne values if 7 come fron	the table ha n the use of	s not been o degrees.	completed.	
<ul> <li>(b)</li> <li>B1: Correct is</li> <li>M1: Correct 2(sum on M0. Folo</li> <li>Note that missing</li> </ul>	Interval width application of f middle valu low through t at $\left\{\frac{"0.5"}{2}\right\} \times$ brackets are i	seen or imp f the trapezines), but if or their values f 1.319 + 2("1)	lied by wor um rule. Mu ne middle va from (a). All 1.075"+1.00 ubsequent w	king. st all be y va lue is omitte low obvious 01+"1.041' vork.	alues, and sh ed allow as a copying slip '+1.223)+1	ould be first a slip. A repe ps. .850 scores	t + last + eated value is s M0 unless t	he

Allow this mark if they add the areas of individual trapezia e.g.

$$\left\{\frac{"0.5"}{2}\right\} \times \left(1.319 + "1.075"\right) + \left\{\frac{"0.5"}{2}\right\} \times \left("1.075" + 1.001\right) + \dots$$

A1: For awrt 2.96. Allow to come from answers in (a) incorrectly rounded or wrong accuracy.FYI: Use of rounded figure gives 2.96225, use of calculated figures gives 2.9625726... Actual value is 2.89 to 2 decimal places.

(c) Condone all work in (c) written without the "10" in  $\log_{10}(\sin(x))$ 

**B1:** Writes  $3 + \log_{10}(\sin(x))$  correctly in terms of  $1 - \log_{10}(\sin(x))$ . May be implied.

M1: Integrates the K in  $K \pm (1 - \log_{10} (\sin(x)))$  to Kx,  $K \neq 3$ , substitutes in the limits 3 and 0.5 and

subtracts on e.g.  $4 \times (3 - 0.5)$  and uses the answer to (b).

**A1ft:** Correct answer awrt 7.04, follow through their 2.96 if given to 3sf or better. **Alt:** 

**B1:** Rearranges integral to find  $\int_{0.5}^{3} \log_{10}(\sin(x)) dx = 2.5 - "2.96"$ . May be implied.

M1: Integrates the 3 and substitutes in given limits and subtracts on e.g.  $3 \times (3 - 0.5)$  and adds their attempt at

 $\int_{0.5}^{5} \log_{10}(\sin(x)) dx$  found using the answer to (b).

A1ft: Correct answer awrt 7.04, follow through their 2.96 if given to 3sf or better.

A common incorrect method in (c):

$$\int_{0.5}^{3} 3 + \log_{10}(\sin(x)) dx = [3x]_{0.5}^{3} + "2.96" = "10.46"$$

Scores no marks as they have not used the integral in part (b)

Attempts to use the trapezium rule again in (c) score no marks

Question	Scheme	Marks
3(i)	E.g. $n = 1$ : $2^3 - 1^3 = 7$ , $n = 2$ : $3^3 - 2^3 = 19$ , $n = 3$ : $4^3 - 3^3 =$ Or identifies counterexample directly	M1
	e.g. $6^3 - 5^3 = 91 = 7 \times 13$ so not true for $n = 5$ , hence statement is not true.	A1
		(2)
	Notes for part (i)	
	<b>M1:</b> Shows evidence of trying to find a counter example for a positive integer (at least one attempt).	
	$2^3 - 1^3$ is prime is sufficient.	
	A1: Gives a correct counter example with reason (shows factorisation) and concludes e.g. "which is not prime". Ignore any previous "incorrect" attempts e.g. $6^3 - 5^3 = 91$ which is prime.	
	Note $n = 7$ (169 = 13×13) and $n = 8$ (217 = 7×31) and $n = 12$ (469 = 7×67) are the next few counter examples. (Bigger examples are not likely to be seen!)	
	Allow equivalent reasons for not being prime e.g. $169/13 = 13$ or $169$ is divisible by 13 (condone "can be divided by 13")	
	Generally algebraic approaches score no marks unless they substitute numbers as indicated above.	
<b>3(ii)</b>	The majority of methods here will follow ways 1, 2 or 3 below	
	In these cases the general guidance is as follows:	
	M1: Attempts to find	
	• the gradient of any relevant line, e.g. AC or BC or	
	• the length of any relevant line, e.g. $AB/AB^2$ or $BC/BC^2$ or $AC/AC^2$ or	
	• the mid-point <i>M</i> of line <i>AB</i>	
	A1: Correct <b>relevant</b> calculation of	
	• gradients AC and BC	
	• lengths of lines $AB/AB^2$ , $BC/BC^2$ and $AC/AC^2$	
	• mid-point of line <i>AB</i>	
	dM1: Full attempt at combining all <b>relevant</b> information required to solve the prob- lem	
	• attempts product of gradients or equivalent	
	• attempts to show Pythagoras $AB^2 = AC^2 + BC^2$	
	• attempts to show $MA^2 = MC^2$	
	A1: Correct calculations or equivalent providing required evidence for the above	
	A1: Provides correct reason and conclusion with all previous marks scored.	

Way 1
$$m_{AC} = \frac{-6-0}{7-1} = \dots$$
 or  $m_{BC} = \frac{-6-(-10)}{7-3} = \dots$ M1 $m_{AC} = -1$  and  $m_{BC} = 1$ A1So  $m_{AC} \times m_{BC} = -1 \times 1 = -1$ dM1A1or e.g.  $m_{AC}$  is negative reciprocal of  $m_{BC}$ dM1A1So e.g. angle (at C) is a right angle hence AB is a diameter  
(Or equivalent)A1

M1: Attempts the gradients of AC or BC. Allow slips but score M0 if both attempts are clearly incorrect.

A1: Correct gradients from correct formulae

**dM1:** Applies perpendicular condition. May be seen as shown but allow equivalent work.

A1: Correct calculations or equivalent

A1: Suitable explanation and conclusion given with no errors and all previous marks awarded with no incorrect statements seen.

Way 2	$AB = \sqrt{(3-1)^2 + (-10-0)^2} = \dots$ or $AC = \sqrt{(7-1)^2 + (-6-0)^2} = \dots$	M1
	or $BC = \sqrt{(7-3)^2 + (-6+10)^2} = \dots$	

$$AB = \sqrt{104} \left( 2\sqrt{26} \right), AC = \sqrt{72} \left( 6\sqrt{2} \right), BC = \sqrt{32} \left( 4\sqrt{2} \right)$$
 A1

$$AB^2 = 104 = 72 + 32 = AC^2 + BC^2$$
 **dM1A1**

 Hence ABC is a right-angle triangle with hypotenuse AB hence AB is a diameter. (Or equivalent)
 A1

 (5)

M1: Attempts length of *AB* or *AC* or *BC* or their squares. Allow slips but score M0 if attempts are clearly incorrect.

A1: Correct values for *AB*, *AC* and *BC* or their squares.

dM1: Applies Pythagoras' theorem with their values. (May see cosine rule used.)

A1: All calculations correct for this approach.

A1: Suitable explanation and conclusion given with no errors and all previous marks awarded with no incorrect statements seen.

Way 3

3 If AB is diameter centre must be midpoint of AB ie 
$$M\left(\frac{1+3}{2}, \frac{0-10}{2}\right)$$
 M1  

$$=(2,-5)$$
A1  
 $MA = \sqrt{(2-1)^2 + (-5-0)^2} = \sqrt{26}, MC = \sqrt{(2-7)^2 + (-5-(-6))^2} = \sqrt{26}$ 
dM1A1  
 $MA = \sqrt{26}, MC = \sqrt{26}$  so  $MA = MC(=MB)$   
As the length from M to each of A and C is the same M is the centre of the circle  
hence AB is a diameter. (Or equivalent)
(5)

M1: Attempts midpoint of *AB*. If no method is shown accept one correct coordinate as evidence. A1: Correct midpoint

**dM1:** Attempts length of *MC* and at least one of *MA* or *MB*, or *AB*. *As M is midpoint of AB there is no need to find both MA and MB, these may be assumed to be the same*. If they find *AB* then they must halve it to find the radius.

A1: All required calculations correct for this approach.

A1: Suitable explanation made which may be in a preamble and conclusion given with no errors and all previous marks awarded.

#### The following approach is less common and should be marked as shown:

Way 4	If <i>AB</i> is diameter centre must be midpoint of <i>AB</i> ie $M\left(\frac{1+3}{2}, \frac{0-10}{2}\right)$	M1
	=(2,-5)	A1
	$MA = r = \sqrt{(2-1)^2 + (-5-0)^2} = \sqrt{26}$	
	$\Rightarrow (x-2)^2 + (y+5)^2 = 26$	dM1A1
	$C(7,-6) \Rightarrow (7-2)^{2} + (-6+5)^{2} = 5^{2} + 1^{2} = 26$	
	As <i>C</i> also satisfies the equation of the circle then <i>AB</i> must be the diameter (or equivalent) There must be some further justification as above rather than just " <i>AB</i> is a diameter" which may be in a preamble e.g. If <i>C</i> lies on the circle	A1
		(5)
M1: Attempt	s midpoint of <i>AB</i> . If no method is shown accept one correct coordinate as evidence.	
AI: Correct	miapoint and substitutes the second property of the single and substitutes the second subs	
coording	ates of C	

A1: All required calculations correct for this approach.

A1: Suitable explanation and conclusion given with no errors and all previous marks awarded.

#### SEE END OF MARKSCHEME FOR SOME OTHER MORE UNUSUAL METHODS THAT MAY BE SEEN DURING MARKING WITH SUGGESTED MARKING GUIDELINES.

There may be other methods. Choose the way that best fits the overall response. If you are in any doubt if a particular response deserves credit then use Review.

Question	Scheme	Marks
4	$\log_4 a + \log_4 b = \log_4 ab$ or $\log_4 ab'' = 3 \Rightarrow ab'' = 4^3$	M1
	$ab = K, a-b = 8 \Longrightarrow a(a-8) = K$ or $(b+8)b = K$	dM1
	$a^2 - 8a - 64 = 0$ or $b^2 + 8b - 64 = 0$	A1
	$a = \frac{-(-8) + \sqrt{(-8)^2 - 4 \times 1 \times -64}}{2 \times 1} = \dots \text{ or } b = \frac{-8 + \sqrt{8^2 - 4 \times 1 \times -64}}{2 \times 1} = \dots$	M1
	$a = 4 + 4\sqrt{5} \Rightarrow b = -4 + 4\sqrt{5}$	M1
	$a = 4 + 4\sqrt{5}$ and $b = -4 + 4\sqrt{5}$ and no other solutions.	A1
		(6)
		(6 marks)
Notes:		
M1: Correct replaces	addition law applied (may be implied) or undoes a log equation correctly, or the 3 by $\log_4 4^3$ oe	
The add	ition law may be seen after substitution from the first equation e.g.	
a-b=	$8 \Longrightarrow a = b + 8 \Longrightarrow \log_4 a + \log_4 b = \log_4 (b + 8) + \log_4 b = \log_4 b (b + 8)$	
Condon	$\log_4(b+8) + \log_4 b = \log_4 b^2 + 8b$	
dM1: Remov	yes logs correctly and proceeds from an equation of the form $ab = K$ and the	
given e	equation $a-b=8$ to a quadratic equation in either a or b.	
A1: Correct of side. The	quadratic equation. Brackets must be expanded but terms not necessarily all on one e "=0" may be implied by their attempt to solve.	
M1: Solves a	3TQ in a or b to obtain a positive solution. There is no need to see the	
negative	e solution in their working.	_
wil: Uses the	er <i>a</i> to find <i>b</i> or vice versa. This depends on having solved a 31Q earlier and obtained	a
root of t	he form $p+q\sqrt{r}$ , $p,q,r \neq 0$ to obtain another root of a similar form i.e. not a decima	al.

The value of a or b does not necessarily have to be positive for this mark. Note that having found a or b they may repeat the process above to find the other value which is acceptable.

A1: Both of  $a = 4 + 4\sqrt{5}$  and  $b = -4 + 4\sqrt{5}$  simplified, with no other solutions. Apply isw once correct answers are seen, e.g. if they subsequently go into decimals.

Question	Scheme	Marks
5	$3\tan(\theta + 43^{\circ}) = 2\cos(\theta + 43^{\circ}) \Longrightarrow 3\frac{\sin(\theta + 43^{\circ})}{\cos(\theta + 43^{\circ})} = 2\cos(\theta + 43^{\circ})$	M1
	$\Rightarrow 3\sin(\theta + 43^\circ) = 2\cos^2(\theta + 43^\circ)$	
	$\Rightarrow 3\sin(\theta + 43^\circ) = 2(1 - \sin^2(\theta + 43^\circ))$	M1
	$\Rightarrow 2\sin^2(\theta + 43^\circ) + 3\sin(\theta + 43^\circ) - 2 = 0$	N/1
	$\Rightarrow (2\sin(\theta + 43^\circ) - 1)(\sin(\theta + 43^\circ) + 2) = 0 \Rightarrow \sin(\theta + 43^\circ) = \dots$	MI
	$\sin\left(\theta + 43^{\circ}\right) = \frac{1}{2}$	A1
	$\theta = \arcsin^{"}\frac{1}{2}^{"}-43^{\circ}$	M1
	$\theta = -13^\circ, \ 107^\circ$	A1
		(6)
		(6 marks)
Notes:		
M1: Uses ta	$n_{} = \frac{\sin}{\sin}$ and multiplies through to form an equation of the form $A \sin = B \cos^2$	

M1: Uses  $\tan ... = \frac{\sin w}{\cos ...}$  and multiplies through to form an equation of the form  $A \sin ... = A$ 

Condone poor notation e.g.:

$$3\tan(\theta + 43^\circ) = 2\cos(\theta + 43^\circ) \Longrightarrow 3\frac{\sin}{\cos}(\theta + 43^\circ) = 2\cos(\theta + 43^\circ)$$

 $\Rightarrow$  3sin( $\theta$  + 43°) = 2cos<sup>2</sup>( $\theta$  + 43°) (with or without brackets)

**M1:** Applies Pythagorean identity to obtain a 3 term quadratic equation in sin. Allow use of  $\cos^2 ... = \pm 1 \pm \sin^2 ...$ 

**M1:** Solves a 3 term quadratic in  $\sin(\theta + 43^\circ)$  by any valid means.

This may be implied by at least one correct root for their quadratic.

Allow if they have  $\sin(\theta + 43^\circ) = x$  or another variable or e.g.  $\sin \alpha$  where  $\alpha = \theta + 43^\circ$ 

A1: Correct value of  $\sin(\theta + 43^\circ)$ . If  $\sin(\theta + 43^\circ) = x$  is used, it must be clear they mean

 $\sin(\theta + 43^\circ)$  but this may be implied if they have e.g.  $\sin \alpha = \frac{1}{2}$  where  $\alpha = \theta + 43^\circ$ 

If  $x = \frac{1}{2}$  is left as a final answer it is A0.

M1: Correct method for solving  $\sin(\theta + 43^\circ) = k$ , |k| < 1, look for use of inverse sine followed by

subtraction of 43 from  $\sin^{-1}(\text{their } k)$ . Implied by one correct solution for their k

Do not allow mixing of degrees and radians for this mark.

A1: Correct solutions and no others in the range.

Question	Scheme	Marks
6(a)	$u_n = ar^{n-1} \Longrightarrow ar + ar^2 = 6$ and $ar^3 = 8$	M1
	$\Rightarrow \frac{ar+ar^2}{ar^3} = \frac{6}{8} \Rightarrow 1+r = \frac{3}{4}r^2$	M1
	$\Rightarrow 3r^2 - 4r - 4 = 0 *$	A1*
		(3)
(a) Way 2	$u_4 = 8 \Longrightarrow u_3 = \frac{8}{r}, \ u_2 = \frac{8}{r^2}$	M1
	$\frac{8}{r} + \frac{8}{r^2} = 6$	M1
	$\Rightarrow 3r^2 - 4r - 4 = 0 *$	A1*
		(3)
(b)	$r = -\frac{2}{3}$	B1
	$ar^3 = 8 \Longrightarrow a = \frac{8}{\left(-\frac{2}{3}\right)^3} = \dots$	M1
	$u_1 = -27$	A1ft
		(3)
(c)	$S_{\infty} = \frac{-27}{1 - \left(-\frac{2}{3}\right)} = \dots$	M1
	$=-\frac{81}{5}$	A1
		(2)
		(8 marks)
Notes: (a) M1: Uses the May also $u_1r + u_1^A$ Must be Alternat M1: Attempt two equ. Allow si	<b>Ignore labelling and mark (a), (b) and (c) together</b> e correct <i>n</i> -th term formula for a GP to set up two equations in <i>a</i> and <i>r</i> to be in terms of $u_1$ and <i>r</i> or $u_2$ and <i>r</i> or $u_3$ and <i>r</i> e.g. $r^2 = 6$ , $u_1r^3 = 8$ or $u_2 + u_2r = 6$ , $u_2r^2 = 8$ or $\frac{u_3}{r} + u_3 = 6$ , $ru_3 = 8$ using the correct term formula so e.g. $ar^2 + ar^3 = 6$ and $ar^4 = 8$ is M0 ively may use the correct sum formula for the first equation, $a\frac{1-r^3}{1-r} - a = 6$ oe. s to solve their two equations to get an equation in <i>r</i> . Look for an attempt to divide the ations, or an attempt to find <i>a</i> in terms of <i>r</i> from one and substitute into the other.	

Alternatively, attempts to eliminate *r* from the equation,  $r = \frac{2}{\sqrt[3]{a}} \Rightarrow \frac{2a}{\sqrt[3]{a}} + \frac{4a}{\sqrt[3]{a^2}} = 6$ 

 $\Rightarrow 2a^{\frac{2}{3}} + 4a^{\frac{1}{3}} = 6$ . Award when they reach a quadratic in  $a^{\frac{1}{3}}$  in this case.

A1\*: CSO Note that as we are marking (a), (b) and (c) together, allow the printed answer to appear anywhere as long as it follows correct work but the "=0" must be seen not implied.

Way 2:

**M1:** Uses  $u_4 = 8$  to write  $u_2$  and  $u_3$  in terms of r

**M1:** Uses  $u_2 + u_3 = 6$  to get an equation in *r* 

A1\*: CSO

(b)

- **B1:** Correct value of *r* seen or used in their working even if subsequently rejected and ignore any other value offered e.g. r = 2
- **M1:** Uses a value for *r* from solving the equation given in (a) where |r| < 1 in one of their equations from (a) to find a value for  $u_1$  or *a*. Allow slips but the algebra should essentially be correct

A1ft: Correct value and no other values. Follow through on  $\frac{8}{(\text{their } r)^3}$  for a value of r with |r| < 1

(c)

**M1:** Uses the correct sum formula with their  $a(u_1)$  obtained from |r| < 1 and r where |r| < 1, to find the sum to infinity.

A1: Correct answer and no other values. Allow equivalents e.g. -16.2

Question	Scheme	Marks
7	$(x+2)$ a factor $\Rightarrow$ f(-2) = 0 $\Rightarrow$ -8A+24+8+B=0	M1A1
	$\int f(x)dx = \frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx$	M1 A1
	$\int_{3}^{5} f(x) dx = 176 \Longrightarrow \left[\frac{A}{4}x^{4} + 2x^{3} - 2x^{2} + Bx\right]_{3}^{5} = 176$	dM1
	$\Rightarrow \left(\frac{A}{4}5^{4} + 2(5^{3}) - 2(5^{2}) + 5B\right) - \left(\frac{A}{4}3^{4} + 2(3^{3}) - 2(3^{2}) + 3B\right) = 176$	uivii
	8A - B = 32 136A + 2B = 12	dM1
	$A = \frac{1}{2}, B = -28$	A1
		(7)
		(7 marks)

#### Notes:

**M1:** Uses the factor theorem or long division (see below) to find one linear **equation** relating *A* and *B*. The "= 0" must be seen or implied by later work.

A1: Correct equation. Award once a correct equation is seen unsimplified or simplified.

So allow e.g.  $A(-2)^3 + 6(-2)^2 - 4(-2) + B = 0$  but not e.g.  $A \times -2^3 + 6 \times -2^2 - 4 \times -2 + B = 0$ 

unless the indices are processed correctly subsequently to e.g. -8A + 24 + 8 + B = 0

The "= 0" must be seen or implied by later work.

M1: Attempts to integrate the given expression, look for at least one power increased by 1. This can also be evidenced by  $B \rightarrow Bx$ 

A1: Correct integration. Allow unsimplified e.g.  $\frac{6x^{2+1}}{2+1}$  for  $2x^3$ 

Condone poor notation e.g. 
$$\int f(x)dx = \int \frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx$$
 and ignore any "+ c"

**dM1:** Applies limits 5 and 3 either way round and subtracts and equates to 176 to obtain an equation in *A* and *B*. **Depends on** <u>previous</u> **M**.

Condone poor use of brackets when subtracting e.g. allow

$$\frac{A}{4}5^4 + 2(5^3) - 2(5^2) + 5B - \frac{A}{4}3^4 + 2(3^3) - 2(3^2) + 3B = 176$$

**dM1:** Solves their two equations in *A* and *B* to find values for both constants.

#### Depends on <u>all previous</u> M marks <u>but allow this mark if the only error was to attempt</u> f(2) = 0 at the beginning.

A1: Correct values.

Note that some candidates eliminate *A* or *B* by rearranging the first equation and substituting into the integral. This essentially follows the main scheme:

$$f(-2) = 0 \Rightarrow -8A + 24 + 8 + B = 0 \Rightarrow B = 8A - 32$$
MIA1
$$\int f(x)dx = \frac{A}{4}x^4 + 2x^3 - 2x^2 + 8Ax - 32x$$
MIA1
$$\Rightarrow \left(\frac{A}{4}5^4 + 2(5^3) - 2(5^2) + 40A - 160\right) - \left(\frac{A}{4}3^4 + 2(3^3) - 2(3^2) + 24A - 96\right) = 176$$
dM1
$$\Rightarrow A = ..., B = ...$$
dM1
$$A = \frac{1}{2}, B = -28$$
A1
$$Long Division for reference:$$

$$x + 2 \left[\frac{Ax^2}{Ax^3 + 6x^2 - 4x + 8} + \frac{B}{Ax^3 + 6x^2 - 4x + 8} + \frac{B}{(6 - 2A)x^2 - 4x + 8} + \frac{B}{(4A - 16)x} + \frac{B}{8A - 32} +$$

Question	Scheme	Marks
<b>8</b> (a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = Ax^3 + B + \frac{C}{x^3}$	M1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 4 \times 256x^3 - 304 - 2 \times 27x^{-3}$ or e.g. $1024x^3 - 304 - \frac{54}{x^3}$	A1A1
		(3)
(b)	$\frac{dy}{dx} = 0 \text{ at SP} \Longrightarrow 1024x^3 - 304 - \frac{54}{x^3} = 0 \Longrightarrow 1024x^6 - 304x^3 - 54 = 0$	M1
	$\Rightarrow 512(x^3)^2 - 152x^3 - 27 = 0 \Rightarrow (8x^3 + 1)(64x^3 - 27) = 0 \Rightarrow x^3 = \dots$	dM1
	$x^{3} = -\frac{1}{8}$ or $x^{3} = \frac{27}{64}$ or $x = -\frac{1}{2}$ or $x = \frac{3}{4}$	A1
	$y = 256 \left( \left\  -\frac{1}{2} \right\  \right)^4 - 304 \left( \left\  -\frac{1}{2} \right\  \right) - 35 + \frac{27}{\left( \left\  -\frac{1}{2} \right\  \right)^2} = \dots \text{ or with } x = \frac{3}{4}$	dM1
	Coordinates are $\left(-\frac{1}{2}, 241\right)$ and $\left(\frac{3}{4}, -134\right)$	A1
		(5)
		(8 marks)
Notes:		
<ul> <li>(a)</li> <li>M1: Attempt twice).</li> <li>A1: At least the A1: Fully contained.</li> </ul>	s the derivative, with at least two terms of the correct form (ie $x^n \rightarrowx^{n-1}$ at least two terms correct, need not be simplified. trect derivative, need not be simplified. Isw after a correct (unsimplified) answer.	
Note that	It there is no need to see the " $\frac{dy}{dx}$ = "in (a) just look for the differentiation.	
Ignore a	ny spurious "= 0"	
(b) M1: Sets the equation Allow e	ir derivative equal to zero and multiplies through by " $x^3$ " to achieve a polynomial a. If in doubt at least 2 terms must be multiplied. quivalent work e.g.	
$1024x^{3}$	$-304 - \frac{54}{x^3} = 0 \Longrightarrow \frac{1}{x^3} (1024x^6 - 304x^3 - 54) = 0 \Longrightarrow 1024x^6 - 304x^3 - 54 = 0$	
To score	e this mark, the derivative must have a negative power of x so allow for e.g.	
$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)$	$1024x^3 - 304 - 54x^{-1} = 0 \Longrightarrow 1024x^4 - 304x - 54 = 0$	
dM1: Solves	a 3 term quadratic in $x^3$ by any valid means (including calculator).	
Must c	ome from an attempt at $\frac{dy}{dx} = 0$ not $\frac{d^2y}{dx^2} = 0$ or $\int y  dx = 0$	
Condo	ne use of inequalities rather than "=" as long as they solve an equation.	

Note that here we accept e.g.  $512(x^3)^2 - 152x^3 - 27 = 0 \Rightarrow (8y+1)(64y-27) = 0 \Rightarrow y = ...$ Or even  $512(x^3)^2 - 152x^3 - 27 = 0 \Rightarrow (8x+1)(64x-27) = 0 \Rightarrow x = ...$ A1: Achieves at least one correct value for x or  $x^3$ . It must be clear that they are values for  $x^3$  here so  $y = \dots$  is acceptable if  $y = x^3$  is seen or implied. If they have  $x = \dots$  then they must have cube rooted. So e.g.  $(8x+1)(64x-27) = 0 \implies x = -\frac{1}{8}$  unless they recover and recognise they have  $x^3$ then this is A0 dM1: Proceeds to find the y coordinate for at least one value of x. Must cube root and not e.g. square root the solution from the quadratic first. The x must have come from a 2 or 3 term quadratic in  $x^3$ . If no method shown, accept any value for y having found an x value but score M0 if there is no evidence of cube rooting or clear evidence that they have substituted into something other than the curve equation but condone if they clearly just mis-copy the equation. Depends on having scored at least 1 of the previous M marks in (b) A1: Both correct pairs of coordinates. Accept if given as e.g.  $x = -\frac{1}{2}$ , y = 241 and  $x = \frac{3}{4}$ , y = -134And no other values. Allow equivalent fractions or values for the  $-\frac{1}{2}$  and  $\frac{3}{4}$ 

Question	Scheme	Marks
9(a)		
	Correct shape from or passing through a point on positive vertical axis. May extend to the left of the vertical axis and allow to pass into quadrant 4. There must be no (obvious) turning points. Labels not required on axes and ignore any that are given.	M1
	Shape and position correct, accept 1 or $k$ as intercept on the positive vertical axis and allow to extend to the left of the vertical axis as shown. Condone (1, 0) or ( $k$ , 0) as long as it is in the correct position. The curve should approach a horizontal asymptote that is half-way between the horizontal axis and the intercept or below. Be tolerant with "wobbles" as it approaches the asymptote. May just "touch" the horizontal axis but not go below it. Labels not required on axes and ignore any that are given.	A1
		(2)
(b)	$\frac{1}{2}k = k\lambda^{5700} \Longrightarrow \lambda^{5700} = \frac{1}{2}$ (see notes for method via substitution)	M1
	$\Rightarrow \lambda = \left(\frac{1}{2}\right)^{\frac{1}{5700}} = 0.999878 \text{ to } 6 \text{ d.p.*}$	A1*
		(2)
(c)	When $t = 3250$ , $N = 15 \times 0.999878^{3250} = \dots$	M1
	= awrt 10.1 (grams)	A1
	10 25 0 000070/	(2) D1
( <b>d</b> )	18 = 25×0.999878	BI
	$\Rightarrow 0.999878^{t} = \frac{18}{25} \Rightarrow t = \frac{\log \frac{18}{25}}{\log 0.999878} = \dots$	M1
	t = 2692.49 so item is 2700 years old	A1
		(3)
		(9 marks)
Notes:		
(a) M1: See sche A1: See sche	eme. me.	

(b)	
<b>M1:</b> Uses the information to set up a correct equation and rearranges to form $\lambda^a = b$ where a	i and $b$
are constants. Allow $\lambda^{5700} = \frac{1}{2}$ to be written down directly.	
A1*: Evidence of taking fractional root seen, leading to the given answer. Alternatively, may	see
logs used, 5700 log $\lambda = \log \frac{1}{2} \Longrightarrow \log \lambda = \frac{-\log 2}{5700} \Longrightarrow \lambda = 10^{\frac{-\log 2}{5700}} = \dots$ Look for at least	one correct
intermediate step (and no incorrect ones). Allow greater accuracy e.g. 0.9998784026	
Note that the use of 5699 instead of 5700 scores M1A0	
Alt:	
M1: Substitutes values 0.9998775 and 0.9998785 or a tighter range containing the root (e.g.	
0.999878 and 0.9998785 will do) to calculate N or just $\lambda^t$ at $t = 5700$ .	
A1*: Correct values, with suitable conclusion that as half value occurs between these $\lambda = 0.9$ to 6 d.p.	99878
(c)	
M1: Substitutes given values (or more accurate $\lambda$ ) into the equation and evaluates. Implied by	' a
correct answer, but an incorrect answer with no evidence is M0. Allow 3249 for 3250.	
Condone the use of a less accurate $\lambda$ e.g. 0.9999 if the intention is clear.	
A1: Awrt 10.1. Units not required. Answer only can score both marks.	
(d)	
B1: Sets up the correct equation from the information given. It must be clear they are using th	ie
given $\lambda$ or better accuracy so clear use of e.g. 0.9999 scores B0 but allow the M1 below.	
This may be implied if they write e.g. $18 = 25 \times \lambda^t$ and then go on to use the given value	of $\lambda$ .
Allow this mark if a different letter is used for t as long as the equation is correct.	
<b>M1:</b> Solves an equation of the form $a = b\lambda^t$ for t to obtain a value	
Must be correct log work so look for e.g. $a = b\lambda^{t} \Rightarrow \frac{a}{b} = \lambda^{t} \Rightarrow t = \log_{\lambda} \frac{a}{b}$	
Or e.g. $a = b\lambda^{t} \Rightarrow \frac{a}{b} = \lambda^{t} \Rightarrow t = \log \frac{a}{b} = \log \lambda^{t} = t \log \lambda \Rightarrow t = \frac{\log \frac{a}{b}}{\log \lambda^{2}}$	
$b$ $b$ $\log \lambda$	
Allow a mis-copied $\lambda$ or a less accurate $\lambda$ for this mark.	
A1: Correct answer of $2/00$ (not awrt). Accept "2/ hundred" or even "2692.49 so $2/"$	
FYI use of $\lambda = 2^{\frac{1}{5700}}$ gives 2701.4 years.	

Question	Scheme	Marks
<b>10(a)</b>	Equation of circle is $(x-3)^2 + (y-5)^2 = r^2$ and line is $y = 2x+k$	M1
	So intersect when $(x-3)^2 + (2x+k-5)^2 = r^2$	
	$\Rightarrow x^{2} - 6x + 9 + 4x^{2} + 4(k - 5)x + (k - 5)^{2} = r^{2}$	dM1
	$\Rightarrow 5x^{2} + (-6 + 4k - 20)x + 9 + k^{2} - 10k + 25 - r^{2} = 0$	uwii
	$\Rightarrow 5x^{2} + (4k - 26)x + k^{2} - 10k + 34 - r^{2} = 0*$	A1*
		(3)
(b)	Tangent to $C \Longrightarrow b^2 - 4ac = 0 \Longrightarrow (4k - 26)^2 - 4 \times 5 \times (k^2 - 10k + 34 - r^2) = 0$	M1
	$\Rightarrow 16k^{2} - 208k + 676 - 20k^{2} + 200k - 680 + 20r^{2} = 0$ $\Rightarrow 5r^{2} = \dots$	M1
	$\Rightarrow 5r^2 = k^2 + 2k + 1 = (k+1)^2$	A1
		(3)
(b) Way 2	Gradient of <i>BX</i> is $-\frac{1}{2}$ so equation of <i>BX</i> is $y-5 = -\frac{1}{2}(x-3)$ $(13-2k \ 26+k)$	M1
	$y-5 = -\frac{1}{2}(x-3), y = 2x+k \Rightarrow x =, y =(5, -5)$	
	$\left(\frac{13-2k}{5}-3\right)^2 + \left(\frac{26+k}{5}-5\right)^2 = r^2$	dM1
	$\Rightarrow 5r^2 = k^2 + 2k + 1 = (k+1)^2$	A1
		(3)
(c)	Triangle AXB is right angled so $AB^2 + r^2 = XA^2 = (3-0)^2 + (5-k)^2$	M1
	$AB^2 = 4r^2 \text{ so } AB^2 + r^2 = 5r^2$	M1
	$\Rightarrow 5r^2 = 9 + (5-k)^2$	A1
	$\implies k^2 + 2k + 1 = 9 + 25 - 10k + k^2$	M1
	$\Rightarrow 12k = 33 \Rightarrow k = \dots$	dM1
	$k = \frac{11}{4}$	A1
		(6)
		(12 marks)
Notes:		
(a) M1. Forma a m	untion of aircle and substitutes in equation of line	
The circle	the equation must be of the form $(x \pm 3)^2 + (y \pm 5)^2 = r^2$	
dM1: Expands	s both sets of brackets and collects terms in $x^2$ and $x$ .	

A1\*: Reaches the answer given with no errors seen.

Note that some candidates expand the brackets first before substitution e.g.:

$(x-3)^2 + (y-1)^2 + (y-1$	$(-5)^{2} = r^{2} \Rightarrow x^{2} - 6x + 9 + y^{2} - 10y + 25 = r^{2} \Rightarrow x^{2} - 6x + 9 + (2x + k)^{2} - 10(2x + k)$	$+25 = r^2$
This imp	lies the first M and then the second M will score when terms in $x^2$ and x are collected.	
Note abo	<b>but poor squaring</b> e.g. $(x-3)^2 = x^2 + 9$ : The first M is available in both cases above bu	t
the secon	d M requires at least two $x^2$ terms and at least two x terms from the expansions.	
Note that $(x-3)^2 + (y-3)^2 + (y-3$	it is acceptable to go from a completely correct full expansion to the printed answer e.g. $(-5)^2 = r^2 \Rightarrow x^2 - 6x + 9 + y^2 - 10y + 25 = r^2 \Rightarrow x^2 - 6x + 9 + (2x + k)^2 - 10(2x + k)$	$+25 = r^2$
$\Rightarrow x^2 - 0$	$5x + 9 + 4x^{2} + 4kx + k^{2} - 20x - 10k + 25 = r^{2} \Longrightarrow 5x^{2} + (4k - 26)x + k^{2} - 10k + 34 - r^{2}$	$r^{2} = 0$
Scores fu	ll marks in (a)	
<b>(b)</b>		
M1: Uses the d	discriminant is zero to form an equation in k and r	
<b>A1</b> . Correct an	s and rearranges to make $\alpha r^{-}$ the subject	
Way 2		
M1: Attempts	the equation of $BX$ and solves simultaneously with $l$ to find the coordinates of $B$	
Alternativ	vely uses $x = -\frac{b}{2a} = \frac{13 - 2k}{5}$ at <i>B</i> and uses this to find <i>y</i> at <i>B</i>	
dM1: Correct	use of Pythagoras for <i>BX</i> and sets = $r^2$	
A1: Correct an	swer	
(c) M1. Attemate	$V_{A}^{2}$ compositive in terms of $k$ (the k-mean energy of the first here real and here $k$ letter) and	
MI: Allempts	<i>Correctly</i> in terms of $k$ (the k may appear as y but must be replaced by k later) and Pythagoras theorem for triangle $AXB$	
May be in	nplied.	
M1: Applies A	$B = 2r$ to get $AB^2 + r^2$ in terms of r. Condone with $AB^2 = 2r^2$ used.	
A1: Correct eq	uation.	
M1: Substitut	es the result from (b) and expands brackets.	
<b>A1:</b> Correct va	linear equation in <i>k</i> . Depends on the previous M.	
	$b = \frac{13-2k}{(13-2k)}$	
Alt (c)	At $B = -\frac{b}{2a} = \frac{15 - 2k}{5}, y = 2\left(\frac{15 - 2k}{5}\right) + k$	M1
	$AB^{2} = \left(\frac{13 - 2k}{5} - 0\right)^{2} + \left(\frac{26 - 4k}{5} + k - k\right)^{2}$	M1 A1
	$\Rightarrow 25 \times 4r^{2} = (13 - 2k)^{2} + (26 - 4k)^{2} = \dots$	M1
	$\Rightarrow 20 \times (k+1)^2 = (13 - 2k)^2 + (26 - 4k)^2 = 20k^2 - 260k + 845$	
	$\Rightarrow k^2 + 2k + 1 = k^2 - 13k + \frac{169}{4} \Rightarrow k = \dots$	dM1
	$\Rightarrow k = \frac{11}{4}$	A1
		(6)

Notes:
(c)
M1: Uses the result in (a) to find the x coordinate where the line and circle meet and then finds y
An alternative is to find the equation of BX as in (b) way 2 and solve with l to find x and y at B
(May have already found the coordinates of $B$ in (a) but must be re-stated or used in (c) to score
this mark)
M1: Uses distance formula to find an expression in k for AB or $AB^2$
A1: Correct expression for $AB$ or $AB^2$ . Need not be simplified.
M1: Applies $AB = 2r$ to the equation. Condone with $AB^2 = 2r^2$ used.
<b>dM1:</b> Substitutes the result from (b) and solves a linear equation in <i>k</i> . <b>Depends on the previous M.</b>
A1: Correct value.

# Some more unusual methods for 3ii

Via perpendicular bisectors:

Γ

Way 5	If <i>AB</i> is diameter centre must be midpoint of <i>AB</i> ie $M\left(\frac{1+3}{2}, \frac{0-10}{2}\right)$	M1
	=(2,-5)	A1
	Attempts 2 of:	
	$m_{BC} = \frac{-6+10}{7-3} = 1$ and midpoint is $\left(\frac{7+3}{2}, \frac{-6-10}{2}\right) = (5, -8)$	
	so perpendicular bisector is $y + "8" = -\frac{1}{"1"}(x - "5")$	
	or	
	$m_{AC} = \frac{7-1}{-6-0} = -1$ and midpoint is $\left(\frac{7+1}{2}, \frac{-6}{2}\right) = (4, -3)$	
	so perpendicular bisector is $y + "3" = -\frac{1}{"-1"}(x - "4")$	dM1
	or	
	$m_{AB} = \frac{-10-0}{3-1} = -5$ and midpoint is $\left(\frac{3+1}{2}, \frac{-10}{2}\right) = (2, -5)$	
	so perpendicular bisector is $y + "5" = -\frac{1}{"-5"}(x - "2")$	
	$y+8 = -(x-5)$ oe or $y+3 = x-4$ oe or $y+5 = \frac{1}{5}(x-2)$ oe	
	And solves simultaneously:	
	E.g. $y+3=x-4, y+8=5-x \Rightarrow 5-x-5=x-4 \Rightarrow x=2, y=-5$	
	Hence centre of circle is $(2, -5)$	A1
	E.g. Midpoint of <i>AB</i> is the centre of the circle so <i>AB</i> is a diameter (or equivalent)	A1
		(5)

M1: Attempts midpoint of *AB*. If no method is shown accept one correct coordinate as evidence.
A1: Correct midpoint
dM1: Attempts 2 perpendicular bisectors, and solves simultaneously
A1: Obtains (2, -5)
A1: Suitable explanation and conclusion given with no errors and all previous marks awarded.

Via circle equation:

Way 6	Uses $(x-a)^2 + (y-b)^2 = r^2$ With (1, 0), (7, -6) and (3, -10) To find $(a, b) = \dots$ or $r/r^2 = \dots$	M1
	Centre $(2, -5)$ or radius $\sqrt{26}$	A1
	E.g. Equation of <i>AB</i> is $y = -5(x-1)$ and $-5(2-1) = -5$	
	or $AB = \sqrt{(3-1)^2 + (-10-0)^2} = \sqrt{104} = 2\sqrt{26}$	dM1A1
	or midpoint of <i>AB</i> is $\left(\frac{1+3}{2}, \frac{0-10}{2}\right) = (2, -5)$	umai
	So centre is on <i>AB</i> or <i>AB</i> is twice the radius or midpoint is the centre hence <i>AB</i> is a diameter of the circle. (or equivalent)	A1
		(5)
M1: Uses al	l three points in circle equation to set up three equations in three unknowns to f	ïnd
centre o	or radius.	
A1: Correct	centre or correct radius	
<b>dM1</b> : Finds	e.g. equation of AB, distance AB or midpoint of AB	
A1: Correct	equation of AB, distance AB or midpoint of AB	

A1: Suitable explanation and conclusion given with no errors and all previous marks awarded.

Via intersecting circles:

(ii) Way 7	If <i>AB</i> is diameter centre must be midpoint of <i>AB</i> ie $M\left(\frac{1+3}{2}, \frac{0-10}{2}\right)$	M1
	=(2,-5)	A1
	Circle centre C radius r is $(x-7)^2 + (y+6)^2 = r^2$	
	Circle centre <i>B</i> radius <i>r</i> is $(x-3)^2 + (y+10)^2 = r^2$	
	These intersect when $(x-7)^2 + (y+6)^2 = (x-3)^2 + (y+10)^2$	
	$\Rightarrow x + y = -3$	dM1A1
	Circle centre A radius r is $(x-1)^2 + y^2 = r^2$	
	$(x-3)^{2} + (y+10)^{2} = (x-1)^{2} + y^{2} \Longrightarrow x - 5y = 27$	

	Solves simultaneously:	
	$x + y = -3$ , $x - 5y = 27 \Longrightarrow x = 2$ , $y = -5$	
	E.g. Midpoint of <i>AB</i> is the centre of the circle so <i>AB</i> is a diameter (or equivalent)	A1
		(5)
M1: Attempts midpoint of AB. If no method is shown accept one correct coordinate as evidence.		
A1: Correc	t midpoint	

**dM1:** Attempts equations of 2 circles with *A*, *B* or *C* as centre with radius *r*, repeats the process for 2 different circles and finds the intersection of both straight lines and solves simultaneously

A1: Correct coordinates of centre

A1: Suitable explanation and conclusion given with no errors and all previous marks awarded.

Pearson Education Limited. Registered company number 872828 with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom